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Energy-Recycling Semi-Active Method for Vibration Suppression with Piezoelectric Transducers

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A novel energy-recycling method is studied that enables effective semi-active vibration suppression with piezoelectric transducers embedded or bonded to a structure. In this method, the energy converted from the mechanical energy of a vibrating structure is collected in the capacitor of a piezoelectric transducer as an electric charge, and to suppress vibration, rather than dissipate the energy, the polarity of the charge is changed according to the state of vibration. With this method, no energy is supplied to the total system of the structure and transducers with shunt circuit, which means that the system is stable. A simple electric circuit and a control law for multiple-degree-of-freedom systems with multiple piezoelectric transducers are proposed for this method based on energy recycling. Numerical simulation of vibration suppression of a truss structure shows that this method is more effective in suppressing vibration than both a semi-active method without energy recycling and that based on the use of an optimally tuned passive system. A preliminary experiment with a truss structure also shows that this method can effectively suppress vibration in an actual structure. However, there was some discrepancy in the experimental results compared to the results of the numerical simulation performed assuming ideal linear characteristics of the piezoelectric transducers estimated from a static test.

Nomenclature

a	=	amplitude of vibration
$\boldsymbol{B}_1$	=	input matrix
b_a	=	electric–mechanical coupling constant of a piezoelectric transducer; Eqs. (1) and (2)
C_a	=	constant-length capacity of a piezoelectric transducer
$\boldsymbol{f}$	=	external force vector
f_a	=	tensile force on a piezoelectric transducer
I_{rms}	=	performance index for vibration suppression; Eq. (36)
$\boldsymbol{K}$	=	stiffness matrix of structure estimated with the shunt circuit open
k_a	=	constant-charge stiffness of a piezoelectric transducer
L	=	inductance
$\boldsymbol{M}$	=	mass matrix
m_1	=	mass shown in Fig. 1
Q	=	electric charge given to a piezoceramic
$\boldsymbol{Q}$	=	vector composed of Q_j
Q_T	=	control input charge for active control, also target charge for semi-active control
$\boldsymbol{q}$	=	vector of modal displacement
q_i	=	ith modal displacement
R	=	electric resistance
u_1	=	x-directional displacement of the tip of truss beam
V	=	voltage applied to a piezoceramic
$\boldsymbol{V}$	=	vector composed of V_j
V_a	=	voltage generated by a piezoceramic
V_b	=	value of V just before the rapid change of V caused by a switching
V_e	=	value of V just after the rapid change of V caused by a switching
V_T	=	control input voltage for active control
$\boldsymbol{W}_1$, $\boldsymbol{W}_2$	=	weighting matrices; Eq. (31)
$\boldsymbol{x}$	=	vector of displacement of structure
x_1	=	elongation of a piezoelectric transducer, also displacement of mass shown in Fig. 1
$\boldsymbol{z}$	=	state vector; Eq. (30)
δ_{rms}	=	root mean square of displacements of truss nodes
ε_i	=	relative error of the ith value
ζ_c	=	damping ratio of electric vibration; Eq. (18)
ω	=	angular frequency of mechanical vibration
ω_c	=	angular frequency of electric vibration; Eq. (17)

Subscript

j	=	jth piezoelectric transducer (for Q, Q_T, V, and C_a)

Introduction

PIEZOELECTRIC devices, embedded or bonded to a mechanical structure, convert mechanical vibration energy into electric energy and electric energy into mechanical energy. Such devices have been widely used by researchers and engineers as actuators, dampers, and transducers for not only active vibration suppression, but also passive and semi-active vibration suppression.[1] In active vibration suppression, the control system sends an input voltage, or charge, to piezoelectric actuators. Although active vibration suppression is quite effective, the total system may become unstable if the control system is improperly designed. In contrast, in passive vibration suppression, the total system is always stable. The simplest method of passive vibration suppression with piezoelectric devices is to shunt each piezoelectric device on a simple resistor. However,

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this method is not very effective for vibration damping. To improve the performance of passive vibration suppression, Hagood and von Flotow,[2] Hagood and Crawley,[3] and Wu[4] proposed to use electric resonance by shunting the piezoelectric transducer on a circuit composed of a resistor and an inductor. However, in practice, to enjoy the advantages of this method, large inductance is usually required at low frequency, which means that a heavy coil must be used.

Several methods of semi-active vibration suppression with piezoelectric devices have also been proposed and studied. In these methods, a resistive, capacitive, or inductive shunt circuit for the piezoelectric transducer embedded or bonded to a structure is switched on and off according to the phase of a vibration mode of structure.[5–9] Semi-active vibration suppression, in this paper, means controlling the states of vibrating structure, piezoelectric devices, and shunt circuit so that its inherent passive vibration damping is enhanced without adding any energy to the system. This method exploits passive energy-dissipation mechanisms of electric resistance, and with it, the total system composed of the structure, transducers, and shunt circuits is not supplied with any additional energy. Therefore, with semi-active vibration suppression, the system is stable even when it is controlled improperly. Although with this method we can usually expect better vibration suppression than with the passive method, semi-active vibration suppression is generally less effective than active vibration suppression.

Vibration suppression means taking energy out of the vibrating structure. Therefore, some researchers have proposed collecting the energy taken out from the vibrating structure when they suppress the vibration.[10–12] We believe that the collected energy can be used to suppress vibration even faster as the work of Lesieutre[12] aimed as a final goal. With no additional energy supplied to the vibrating system, the vibration energy and the collected energy dissipate. This method can be called semi-active vibration suppression, and with it, the system is stable, as shown subsequently in the Appendix. In this paper, this method is referred to as energy-recycling vibration suppression.

To semi-actively suppress the vibration, Richard et al.[7] proposed to shunt a piezoelectric transducer on an inductive circuit for a short time at each peak of vibration strain. During this short shunting, the polarity of the charge in the capacitor of piezoelectric transducer is reversed such that the charge drives the transducer against the vibration. In this method, the electric energy stored in the capacitor is recycled rather than immediately dissipated. Therefore, this method is an energy-recycling method although it is not clearly mentioned. Corr and Clark[8] also showed high-performance of this method in suppressing vibration. However, their method is applicable only to a single-mode vibration. In this paper, we also use the piezoelectric transducers with inductive shunt circuits to implement an energy-recycling vibration suppression. Based on an energy-recycling concept, a switching method of inductive shunt circuit for the piezoelectric transducer is developed for semi-active vibration suppression. It is applicable to multiple-mode vibration of structures with multiple transducers. The method of Richard et al.[7] and Corr and Clark[8] is shown to be a particular case of our method. The performance of the method is studied theoretically and numerically. To see that this method works in an actual structure, a preliminary vibration suppression experiment of a truss structure is performed.

Semi-Active Vibration Suppression of Single-Degree-of-Freedom System with a Piezoelectric Transducer

To describe the new method of semi-active vibration suppression based on energy-recycling in comparison with a conventional semi-active method, let us first consider a simple single-degree-of-freedom system shown in Fig. 1, which is composed of a mass and a piezoelectric transducer. Assuming that the local dynamics in the piezoelectric transducer are negligible because of its very high frequency, we model the characteristics of a piezoelectric transducer composed of attached fittings and a mechanical member exerting precompression force on the piezoceramic, as shown in Fig. 1. In this paper, we further assume a linear behavior of the piezoceramic.[13] Then the relationship between Q, V, f_a, and x_1 of the transducer can be written as

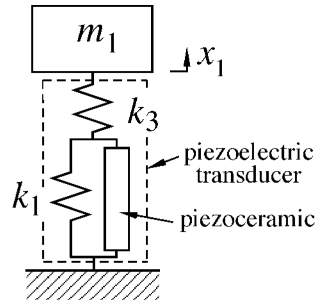


Fig. 1 Single-degree-of-freedom system with a piezoelectric transducer.

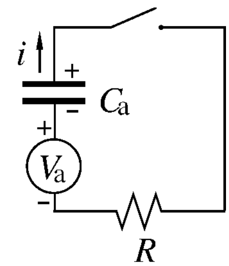


Fig. 2 Circuit A for shunting the piezoelectric transducer.

$$f_a = k_a x_1 - b_a Q \tag{1}$$

$$V = -b_a x_1 + Q/C_a \tag{2}$$

The values of k_a, b_a, and C_a are functions of the size and characteristics[13] of the piezoceramic and the spring constants k_1 and k_3 shown in Fig. 1. From Eq. (1), the equation of motion of this system can be derived as

$$m_1 \ddot{x}_1 + k_a x_1 = b_a Q \tag{3}$$

If we use the active method, we can suppress the vibration of the structure by supplying control input charge Q_T or control input voltage $V_T = -b_a x_1 + Q_T/C_a$ to the piezoelectric transducer according to the state of the system. There are many active control theories that can be used to obtain the value of Q_T. A simple active control law is

$$Q_T = -\alpha \dot{x}_1 \tag{4}$$

where α is the control gain ($\alpha > 0$). Generally, the larger the value of the control gain, the higher the vibration suppression performance is unless the gain is too large.

In this study however, we try to semi-actively suppress the vibration of structure by controlling the state of the passive system instead of actively supplying a control charge or voltage to the transducer.

Resistance (R)-Switching Method for Conventional Semi-Active Vibration Suppression

To compare with the new method of vibration suppression based on energy recycling, let us first describe a conventional semi-active vibration suppression that does not recycle the energy. To implement the conventional method, we shunt the piezoelectric transducer by using a circuit composed of a switch and a resistor, as shown in Fig. 2, and turn the switch on and off so that the vibration is suppressed quickly. Although a small amount of electric power is required for the logic circuit and drive circuit for the switch, this energy does not flow into the shunt circuit. In Fig. 2, the piezoelectric transducer is modeled by using a capacitor and a voltage generator based on Eq. (2), where

$$V_a = -b_a x_1 \tag{5}$$

To derive a control law for switching, let us follow the strategies described in Ref. 14 that were used to derive control laws to suppress vibration with variable-friction devices. As suggested in Ref. 14, a possible strategy to control the switch is to turn the switch on and off so that Q traces Q_T as closely as possible. However, as already mentioned, in many cases, a large gain results in quick vibration damping. Therefore, in this study, we control the switch so that Q becomes as large, that is, positive, as possible when Q_T is positive, and as small, that is, negative, as possible when Q_T is negative. The study in Ref. 14 has shown that this strategy is more effective than tracing Q_T, although the difference between their performances is small. For the system shown in Fig. 2, it is clear that, when the switch is closed,

$$\dot{Q} = -V/R \tag{6}$$

and when the switch is open,

$$\dot{Q} = 0 \tag{7}$$

Therefore, this strategy is implemented by using the following switching law 1:

Turn on the switch when

$$Q_T V < 0$$

Turn off the switch when

$$Q_T V > 0$$

Note that any active control theory can be used to obtain Q_T. In this paper, this semi-active method is referred to as resistance (R)-switching vibration suppression even when a different active control law is used to derive the target charge Q_T. As will be described later, by using a well-established active control theory for a multiple-degree-of-freedom (MDOF) system with multiple inputs to obtain Q_T, we can easily use this method for MDOF systems with multiple transducers. In the case of suppressing the vibration of a single-degree-of-freedom system with a single transducer by using Eq. (4), this R-switching method is the same as that used in Ref. 5. Although another switching method for resistive shunting is proposed in Ref. 6, its performance in suppressing vibration is not as good as that of the method of Ref. 5 in a comparison reported in Ref. 9. In this study, therefore, we compare the performance of our energy-recycling method with that of R-switching, which does not recycle the energy.

To understand how semi-active control based on R-switching works, let us assume that the amplitude of vibration of the system in Fig. 1 is almost constant even when this semi-active control is applied, and let us approximate the motion of the mass as

$$x_1 = a\cos(\omega t) \tag{8}$$

Let us further assume that, when the switch is turned on, the charge in the piezoelectric transducer is discharged in a much shorter period of time than the period of mechanical vibration, assuming that

$$RC_a\omega \ll 1 \tag{9}$$

If we apply switching law 1 with Eq. (4) under these conditions, we can see from Eq. (2) and the earlier mentioned assumptions that the switch is closed for a short time periodically at $t = n\pi/\omega$ and that when $2n\pi - \pi < \omega t < 2n\pi$ the values of V and Q become approximately

$$V = -b_a a(1 + \cos\omega t) \tag{10}$$

$$Q = -C_a b_a a \tag{11}$$

and when $2n\pi < \omega t < 2n\pi + \pi$

$$V = b_a a(1 - \cos\omega t) \tag{12}$$

$$Q = C_a b_a a \tag{13}$$

where n is an arbitrary integer. From these equations, the energy that dissipates in a cycle of vibration is derived as $4C_a b_a^2 a^2$.

Inductance–Resistance (LR)-Switching Method for Energy-Recycling Vibration Suppression

To implement semi-active vibration suppression based on energy recycling, let us shunt the piezoelectric transducer on a circuit composed of a switch, a resistor, and an inductor as shown in Fig. 3.

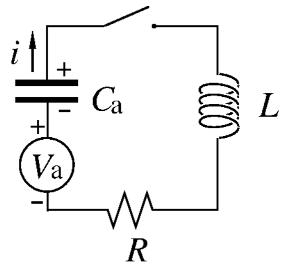


Fig. 3 Circuit B for shunting the piezoelectric transducer.

From Fig. 3, we can see that, when the switch is closed,

$$L\ddot{Q} + R\dot{Q} + Q/C_a = b_a x_1 \tag{14}$$

and, when the switch is open,

$$\dot{Q} = 0 \tag{15}$$

When Eq. (2) is used, Eq. (14) can be rewritten as

$$L\ddot{Q} + R\dot{Q} = -V \tag{14'}$$

As we did in the R-switching method, let us again make the value of Q as large (positive) as possible when Q_T is positive, and as small (negative) as possible when Q_T is negative. It is clear from Eqs. (14′) and (15) that, when the switch in Fig. 3 is turned from open to closed, Q starts to increase if $V < 0$ and to decrease if $V > 0$. Therefore, this method can be implemented by using the following switching law 2:

Turn on the switch when $Q_T V < 0$, and turn it off when $Q_T \dot{Q} < 0$.

To understand the behavior of the structure and control system with this method, let us again use Eqs. (8) and (9). For simplicity, let us further assume that $Q = 0$ at $t = 0$. Then, we can see from Eqs. (2) and (8) that $V < 0$ and from Eqs. (4) and (8) that the value of Q_T turns from negative to positive at $t = 0$. This means that the switch is turned on according to switching law 2 at $t = 0$. Therefore, let us investigate the subsequent change of the value of Q caused by this switching. Based on the assumption in Eq. (9), we assume that the displacement of the mass during a very short period when the value of Q changes is small and that the displacement can be approximated as $x_1 = a$. Then, we can derive the following equation from Eq. (14):

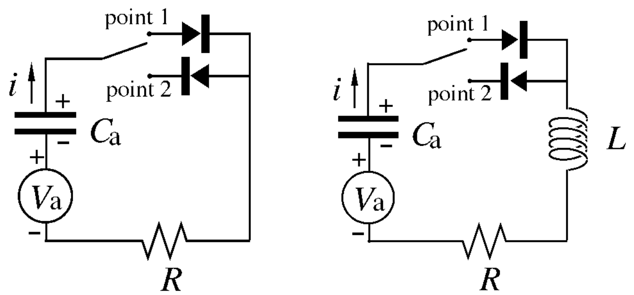
$$Q = -b_a a C_a\Big\{\cos\big[\omega_c(1-\zeta_c)^{\frac{1}{2}}t\big] + \zeta_c(1-\zeta_c)^{-\frac{1}{2}}\sin\big[\omega_c(1-\zeta_c)^{\frac{1}{2}}t\big]\Big\}e^{-\zeta_c\omega_c t} + b_a a C_a \tag{16}$$

where

$$\omega_c = [1/(LC_a)]^{\frac{1}{2}} \tag{17}$$

$$\zeta_c = R/(2L\omega_c) \tag{18}$$

For simplicity, let us further assume that $\zeta_c \ll 1$. We can see that, after the switching at $t = 0$, the value of Q increases to approximately $b_a a C_a(1 + e^{-\zeta_c\pi})$ at $t \approx \pi/\omega_c$. After that, the value of Q starts to decrease. Therefore, according to the switching law, the switch is turned off at this moment, and the value of Q is kept constant. After half a cycle of mechanical vibration, that is, at $t \approx \pi/\omega$, the switch is again turned on because the value of $Q_T V$ becomes negative at this moment. In a similar way, we can see that the value of Q decreases to approximately $-b_a a C_a(1 + 2e^{-\zeta_c\pi} + e^{-2\zeta_c\pi})$ because the initial value of Q is approximately $b_a a C_a(1 + e^{-\zeta_c\pi})$, and the switch is turned off at this moment. Further investigation reveals that the transducer absorbs energy of $2C_a b_a^2 a^2(1 + e^{-\zeta_c\pi})(2 + e^{-\zeta_c\pi})$ from the vibrating mass by the end of the first cycle of the mechanical vibration, that is, by $t \approx 2\pi/\omega$. The absolute value of Q increases at every switching performed once per half a cycle of mechanical vibration. This means that the energy of mechanical vibration is converted into electrical energy and collected in the capacitor of the piezoelectric



a) Circuit A′ for R switching **b) Circuit B′ for LR switching**

Fig. 4 Circuits A′ and B′ for simplified switching law.

transducer. After many cycles of vibration, the value of Q converges to approximately $\pm b_a a C_a(1+e^{-\zeta_c\pi})/(1-e^{-\zeta_c\pi})$, and energy of $4C_a b_a^2 a^2(1+e^{-\zeta_c\pi})/(1-e^{-\zeta_c\pi})$ dissipates per a cycle of vibration. These values of charge and energy are, respectively, larger than $C_a b_a a$ and $4C_a b_a^2 a^2$ estimated for the R-switching method, especially when ζ_c is small. Therefore, we can expect for this method to suppress the vibration more effectively than R-switching method does, especially when the value of ζ_c is small. In this paper, this new semi-active method with the circuit shown in Fig. 3 is referred to as inductance–resistance (LR)-switching energy-recycling vibration suppression even when a different active control strategy is used to derive the target charge Q_T.

Simplifying LR-Switching

In this section, two simplifications of the LR-switching method are described. The first one is based on that, as mentioned in the preceding section, the value of Q, which starts to change when the switch in Fig. 3 is turned on, continues to change for a period of approximately π/ω_c, and the polarity of $\dot{Q}$ changes from positive to negative or from negative to positive at the end of this period. Based on this fact, switching law 2 can be simplified as follows, which is referred to as switching law 3 in this paper:

Turn on the switch for a period of π/ω_c when $Q_T V < 0$.

In this case, we do not need to measure the electric current $\dot{Q}$ to control the switch. In the case of vibration suppression of a single-degree-of-freedom system with a single transducer by using Eq. (4), this switching law is the same as that proposed by Richard et al.[7]

The switching law can be further simplified by using diodes in the shunt circuit, as shown in Fig. 4b. For this circuit, switching law 3 can be further simplified as the following switching law 4:

Turn the switch to 1 when $Q_T < 0$ and to 2 when $Q_T > 0$.

Note that neither current $\dot{Q}$ nor voltage V needs to be measured for this switching law. If we use the shunt circuit shown in Fig. 4a, switching law 4 enables semi-active vibration suppression based on R-switching.

Semi-Active Vibration Suppression of an MDOF System with Multiple Piezoelectric Transducers

In the semi-active vibration suppression methods discussed in the preceding sections, the switch in a passive circuit is controlled so that Q follows the polarity of control input Q_T obtained from an active control theory. Because there are many active control theories for MDOF systems with multiple actuators, these semi-active vibration suppression methods can be easily applied to MDOF systems with multiple piezoelectric transducers by using any one of these active control theories. In this section, the R-switching and LR-switching methods are used for MDOF systems with multiple piezoelectric transducers.

When the local dynamics in piezoelectric transducers are negligible, the equation of motion of a structure with these piezoelectric transducers is written as

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{f} + \boldsymbol{B}_1\boldsymbol{Q} \tag{19}$$

and the voltage of the piezoelectric transducers is given by

$$\boldsymbol{V} = -\boldsymbol{B}_1^T\boldsymbol{x} + \boldsymbol{C}^{-1}\boldsymbol{Q} \tag{20}$$

where $\boldsymbol{C}$ is defined as

$$\boldsymbol{C} \equiv \text{diagonal}\left[C_{a_1}, C_{a_2}, \ldots, C_{a_{n_p}}\right] \tag{21}$$

After the transformation of Eq. (19) into modal coordinates by using

$$\boldsymbol{x} = \boldsymbol{\Phi}\boldsymbol{q} \tag{22}$$

and the introduction of damping ratio ζ for all vibration modes, the equation of motion can be rewritten as

$$\ddot{\boldsymbol{q}} + \boldsymbol{\Xi}\dot{\boldsymbol{q}} + \boldsymbol{\Omega}\boldsymbol{q} = \boldsymbol{\Phi}^T\boldsymbol{f} + \boldsymbol{\Phi}^T\boldsymbol{B}_1\boldsymbol{Q} \tag{23}$$

where

$$\boldsymbol{\Phi} \equiv [\phi_1, \phi_2, \ldots, \phi_m] \tag{24}$$

$$\boldsymbol{\Omega} \equiv \text{diagonal}\left[\omega_1^2, \omega_2^2, \ldots, \omega_m^2\right] \tag{25}$$

$$\boldsymbol{\Xi} \equiv \text{diagonal}[\zeta\omega_1, \zeta\omega_2, \ldots, \zeta\omega_m] \tag{26}$$

and ω_k^2 and ϕ_k are the kth eigenvalue and eigenvector of the following eigenproblem:

$$(\omega^2\boldsymbol{M} + \boldsymbol{K})\phi = 0 \tag{27}$$

where the eigenvectors are normalized so that

$$\phi_k^T\boldsymbol{M}\phi_k = 1 \tag{28}$$

Then the equation of motion in Eq. (23) can be further rewritten as

$$\dot{z} = \boldsymbol{A}z + \boldsymbol{D}\boldsymbol{f} + \boldsymbol{B}\boldsymbol{Q} \tag{29}$$

in terms of the state vector defined as

$$z \equiv \lfloor \boldsymbol{q}^T, \dot{\boldsymbol{q}}^T \rfloor^T \tag{30}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{D}$ are appropriate matrices. If we regard variable $\boldsymbol{Q}$ in Eq. (29) as a control input from an active control system, the linear quadratic regulator (LQR) control theory tells us that the optimal control input that minimizes the performance index

$$J \equiv \int \left(z^T\boldsymbol{W}_1 z + \boldsymbol{Q}^T\boldsymbol{W}_2\boldsymbol{Q}\right) \mathrm{d}t \tag{31}$$

is given[15] as

$$\boldsymbol{Q} = \boldsymbol{Q}_T \equiv -\boldsymbol{F}z \tag{32}$$

$$\boldsymbol{F} = \boldsymbol{W}_2^{-1}\boldsymbol{B}^T\boldsymbol{P} \tag{33}$$

where $\boldsymbol{P}$ is a positive definite solution of

$$\boldsymbol{P}\boldsymbol{B}\boldsymbol{W}_2^{-1}\boldsymbol{B}^T\boldsymbol{P} - \boldsymbol{A}^T\boldsymbol{P} - \boldsymbol{P}\boldsymbol{A} - \boldsymbol{W}_1 = 0 \tag{34}$$

If measuring the values of z is difficult, these values are usually estimated by an observer.[15] When the estimated value of z is used, the active control input can be obtained as

$$\boldsymbol{Q} = \boldsymbol{Q}_T \equiv -\boldsymbol{F}\hat{z} \tag{35}$$

where $\hat{z}$ is the estimated value of z. Once the value of $\boldsymbol{Q}_T$ is obtained, we can control the switch in the shunt circuit for the ith piezoelectric transducer based on Q_{T_i} according to the already described control strategies to enable LR-switching- or R-switching-based semi-active vibration suppression of an MDOF system with multiple piezoelectric transducers.

Numerical Simulation and Discussion

To compare the performance of the LR-switching energy-recycling vibration suppression method with that of the R-switching semi-active vibration suppression method as well as that of passive

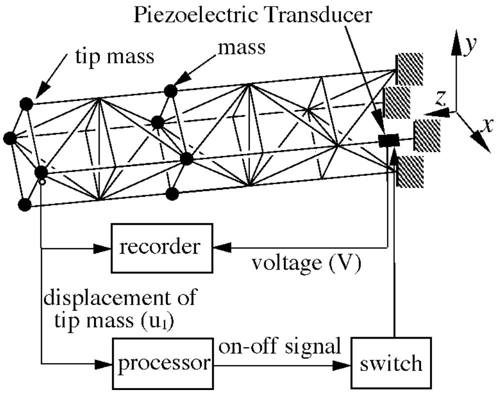


Fig. 5 Five-bay truss structure with a piezoelectric transducer and experimental setup.

vibration suppression, we simulated the vibration suppression of a cantilevered five-bay truss beam shown in Fig. 5. This truss beam was also used in our experiment, which will be described later. Figure 5 also shows a block diagram of the experimental setup. The total length of the truss beam was 1.86 m. A 0.5-kg mass was mounted at each of the four tip nodes and four central nodes of the beam truss. The axial stiffness of the truss was 1.99×10^6 N. The mass of the longitudinal member and that of the lateral member were both 35.7 g, and the mass of the diagonal member was 46.3 g. The mass of each node was 67.9 g.

The values of k_a, b_a, and C_a of the transducer were assumed to be $k_a = 6.17 \times 10^6$ N/m, $b_a = 3.69 \times 10^5$ N/C, and $C_a = 1.20 \times 10^{-5}$ F based on the values estimated experimentally for a Tokin ASB171C801NP0 piezoelectric transducer as will be described later. The damping ratio of each mode was assumed to be 0.5%

Vibration Suppression with a Single Transducer

To understand how the energy-recycling method works, we first performed a numerical simulation of vibration suppression of the truss with a single piezoelectric transducer shown in Fig. 5. All vibration modes that were symmetrical with respect to the x–z plane were taken into account in the mathematical model. All other modes were ignored so that the system was controllable.

After the modal displacement of the first symmetrical mode was set to 1.0 mm kg$^{1/2}$ and the modal displacements of all of the other modes were set to zero, the structure was released, and the subsequent vibration was suppressed by using the piezoelectric transducer with the switching laws described earlier. To measure the vibration suppression performance of the switching laws, the value of

$$I_{\rm rms} \equiv \int_0^{t_E} \delta_{\rm rms}\, {\rm d}t \tag{36}$$

was calculated, where $t_E = 0.5$ s.

To derive the value of Q_T from the LQR control theory for both the R-switching and LR-switching methods, the lowest mode was controlled and weighting matrix $\boldsymbol{W}_1$ in Eq. (31) was set to

$$\boldsymbol{W}_1 = {\rm diagonal}\left[1,\ 1\big/\omega_1^2\right] \tag{37}$$

$\boldsymbol{W}_2$ is a scalar because there is only one transducer in this case. Here, the state vector was assumed to be known, and an observer was not used.

The equations of motion in the modal coordinates were integrated using the Runge–Kutta scheme with the double-precision option. The time step for the integration was 10^{-7} s, which is much shorter than the period of the highest mechanical and electrical vibration mode (0.00049 s). At the moment of release of the structure, the value of electric charge Q was assumed to be zero. The values of L and $\boldsymbol{W}_2$ were roughly optimized so that the value of $I_{\rm rms}$ was minimal when $R = 0.5\ \Omega$, although the value of $I_{\rm rms}$ is relatively insensitive to these values. The resulted optimal values were $\boldsymbol{W}_2 = 1.0 \times 10^3$ kg for R-switching and $L = 8.0 \times 10^{-4}$ H and $\boldsymbol{W}_2 = 1.0 \times 10^3$ kg for LR-switching.

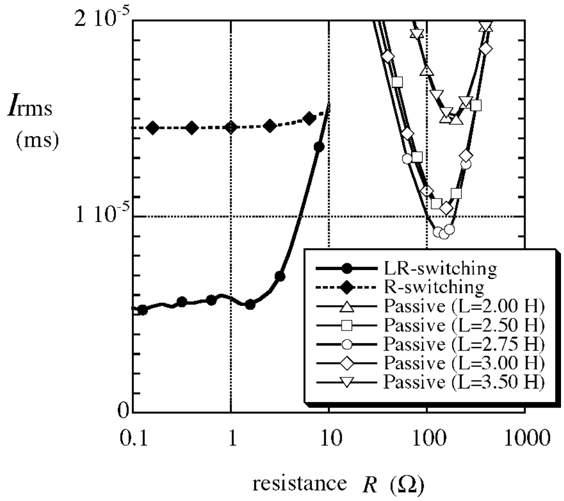


Fig. 6 Vibration suppression performances estimated in numerical simulation for a truss with a single transducer.

To compare with the semi-active control method, we also calculated the value of $I_{\rm rms}$ for a passive system studied by Hagood and von Flowtow[2] and Hagood and Crawley.[3] This passive system was implemented by shunting the electrodes of a piezoelectric transducer by using a circuit composed of an inductor and a resistor. In other words, this passive system was implemented by keeping the switch in Fig. 3 always closed. When the values of L and R were optimized for the system, the electric circuit effectively eliminated the energy of the system as a result of its resonance, just as with mechanical dynamic vibration absorbers.

Figure 6 shows the values of $I_{\rm rms}$ for a system with R-switching semi-active control, that with LR-switching energy-recycling control, and that for the mentioned passive system. Figure 6 shows that the performance of the passive system sensitively depends on the values of L and R. The optimal values of L and R are approximately $L = 2.75$ H and $R = 150\ \Omega$. At these optimal values, the value of $I_{\rm rms}$ for the passive system is lower than that for the system with R-switching semi-active control. Because smaller values of $I_{\rm rms}$ indicate better vibration suppression, this means that, in this particular case, the performance of the optimally tuned passive system is higher than that of the system with R-switching semi-active control. However, note that the optimal value of inductance, 2.75 H, corresponds to too heavy a coil to be practical in suppressing the vibration of a 7.9-kg truss structure.

The value of $I_{\rm rms}$ for the system with R-switching control is insensitive to resistance R. It is larger than the minimum value of $I_{\rm rms}$ for the passive system, but lower than the value of $I_{\rm rms}$ when inductance L is less than 1 H. When resistance R is less than 3 Ω, the value of $I_{\rm rms}$ for the system with LR-switching energy-recycling semi-active control is much lower than that for the system with R-switching semi-active control and the minimum value for the passive system. This means that the system with LR-switching semi-active control suppressed vibration better than the system with R-switching semi-active control and the optimally tuned passive system did.

Figure 7 shows time histories obtained by using the R-switching and LR-switching methods with switching law 4 with $R = 1.0\ \Omega$. In both cases, the switching is performed at (or around) the peaks of the first modal displacement. The value of electric charge Q is indirectly controlled by the switching so that its absolute value is the maximum when its polarity is equal to Q_T and the minimum in all other cases, just as the control strategy specifies. At each switching, both electric charge Q and voltage V of the transducer alternately take positive and negative values following the polarity of Q_T. Figure 7 shows that these values changed more with LR-switching than with R-switching. For example, at the switching at $t = 0.02$ s, the voltage shifted from approximately 24 to −18 V with LR-switching and from approximately 18 to 0 V with R-switching. This difference is a result of inductance in circuit B′ in Fig. 4b. As a result, with LR-switching, the absolute value of Q is kept large, and the vibration is suppressed more quickly than with R-switching. Figure 7 shows

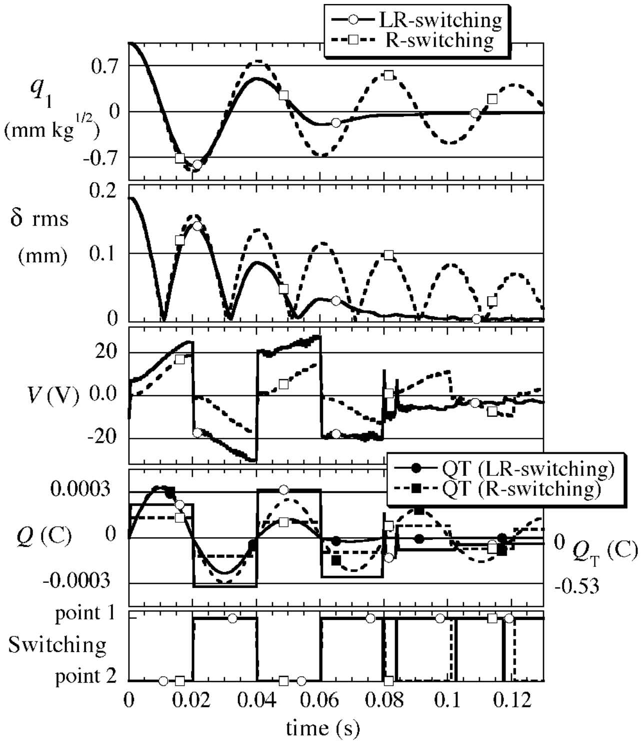


Fig. 7 Time histories of vibration suppression with LR-switching energy-recycling and R-switching semi-active methods: $R = 1.0$ Ω, $L = 8.0 \times 10^{-4}$ H, and $W_2 = 1.0 \times 10^3$ kg.

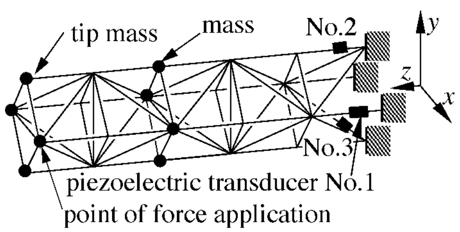


Fig. 8 Five-bay truss structure with three piezoelectric transducers.

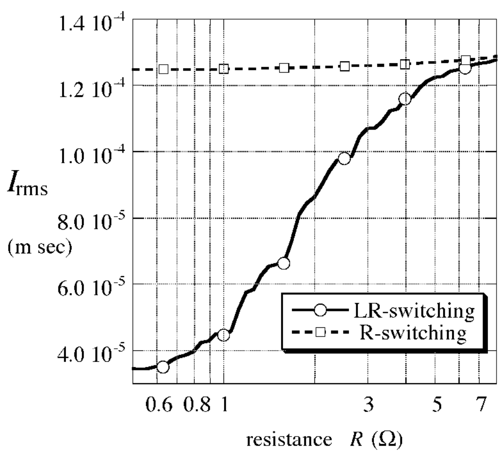


Fig. 9 Vibration suppression performances estimated for a truss with three transducers in the numerical simulation.

that the energy stored in the capacitor is automatically discharged by irregular switching after $t = 0.08$ s when the vibration has been suppressed to a sufficiently small amplitude. This may be because the vibration amplitude becomes too small for the amount of stored energy.

Higher vibration modes can be excited by switching, although they are not shown in Fig. 7. However, as their negligible effects on δ_{rms} in Fig. 7 indicate, their amplitudes are small.

Vibration Suppression with Multiple Transducers

In the preceding section, the LR-switching energy-recycling semi-active method was shown to be effective in suppressing the vibration of a structure with a single transducer. To investigate whether this method is effective in suppressing multiple-mode vibration of a system with multiple transducers, we simulated vibration suppression of the five-bay truss with three piezoelectric transducers, shown in Fig. 8. In this simulation, all of the vibration modes were taken into account in the mathematical model. For simplicity, all of the transducers and electric circuits were assumed to be identical. When the structure was still, a velocity of (1, 1, 1) m/s in the x–y–z coordinates was given to the node shown in Fig. 8 by an impulsive force, and the subsequent free vibration was suppressed. For comparison, we simulated vibration suppression with R-switching. To obtain Q_T for R-switching and LR-switching, the lowest six modes were assumed to be controlled in designing the LQR control law, and the weighting matrices of Eq. (31) were set to

$$W_1 = \text{diagonal}\left[1, \ldots, 1, 1/\omega_1^2, \ldots, 1/\omega_6^2\right] \tag{38}$$

$$W_2 = 1.0 \times 10^3 \text{ diagonal}[1, 1, 1] \tag{39}$$

based on the roughly estimated optimal value of W_2 for the structure with a single transducer described in the preceding section.

The values of I_{rms} obtained for the truss with three transducers with the energy-recycling and conventional semi-active methods are shown in Fig. 9 as a function of resistance R. Figure 9 shows that when R is small, the LR-switching energy-recycling method gives a much smaller value of I_{rms} than the R-switching semi-active method does. This indicates that similarly to the case of a single transducer, the LR-switching method works well and can suppress vibration of a MDOF structure with multiple transducers better than the R-switching method. Thus, LR-switching is effective in suppressing vibration especially when R is small, as was suggested in the preceding section.

Preliminary Experiment of Semi-Active Vibration Suppression

To see whether the LR-switching method is effective for an actual structure with a piezoelectric transducer, we performed a preliminary experiment by using the five-bay cantilevered beam truss shown in Fig. 5. This is the same truss beam used in the simulation described earlier. Figure 5 also shows a block diagram of the experimental setup. The coil inductance in this experiment was 6.81×10^{-4} H, and the total resistance of the coil and harness was 0.76 Ω. In this experiment, circuits A′ and B′ in Fig. 4 were used. The electric current flows through a diode only when the applied voltage is larger than the threshold called forward voltage. To avoid the effect of forward voltage, which is not negligible compared with the operational voltage of the transducer in this experiment, an electric circuit shown in Fig. 10 was used instead of a diode. The equivalent forward voltage of the circuit was less than 5 mV, and the resistance of this device was 0.02 Ω.

A commercially available Tokin ASB171C801NP0 piezoelectric transducer was used in this experiment. The length of the transducer was 0.22 m, and the mass was 93 g. It had a stack of 1300 layers of piezoelectric ceramics. To measure the characteristics of this transducer, we measured the load-elongation relationship with the

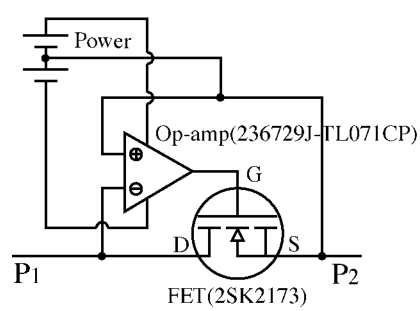


Fig. 10 Circuit simulating a low-forward-voltage diode.

electrodes shorted and open, and the voltage/elongation relationship under load-free conditions, as well as measured the capacitance under load-free conditions. Although the load/elongation and voltage/elongation relations were hysteretic, the values of the constant-charge load/elongation ratio, constant-voltage load/elongation ratio, constant-load elongation/voltage ratio, and constant-load capacitance were determined from the measured data by using the least-square method. From Eqs. (1) and (2), we can see that these values are k_a, $k_a - b_a^2 C_a$, $b_a C_a/(k_a - b_a^2 C_a)$, and $k_a C_a/(k_a - b_a^2 C_a)$, respectively. Because we had four determined values for three parameters, assuming that these measured values had some errors and the ith determined value was $1 + \varepsilon_i$ times the correct ith value, the values of k_a, C_a, b_a were estimated so that $\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2$ was minimal. The estimated values were $k_a = 6.17 \times 10^6$ N/m, $b_a = 3.69 \times 10^5$ N/C, and $C_a = 1.20 \times 10^{-5}$ F, and the minimum value of $\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2$ was 0.00265.

In the experiment, the lowest 23.1-Hz vibration mode in the x–z plane was first excited by using a permanent magnet and a voice coil. The subsequent free vibration of the mode was then semi-actively suppressed by using the R-switching and LR-switching methods. The tip displacement u_1 was measured by using an eddy-current displacement sensor and fed to the processor as shown in Fig. 5. Based on the estimated characteristics of the transducer and the same finite element method model of the truss used for the numerical simulation, the gain matrix $\boldsymbol{F}$ was obtained from Eq. (33). Because both R-switching and LR-switching method watch only the polarity of the elements of $\boldsymbol{Q}_T$, $\lfloor u_1, \dot{u}_1 \rfloor^T$ was used instead of $z = \lfloor q_1, \dot{q}_1 \rfloor^T$ in Eq. (32) assuming that the lowest modal displacement q_1 was proportional to u_1. The value of $\dot{u}_1$ was obtained by differentiating u_1. No observer was used. The values of $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ were the same as those used for the numerical simulation.

Figure 11 shows an example of the time histories obtained in the experiment with LR-switching. Before the start of semi-active vibration suppression, the value of V was oscillating around a positive value because the switch in circuit B′ was kept at 2 while the structure was vibrating freely. We began to suppress vibration at $t = 0.32$ s, and the first-mode vibration was quickly suppressed in the following 0.4 s. During this time, the circuit was switched from 1 to 2 and from 2 to 1 at each of the peaks of the tip-node displacement, and the value of V changed from positive to negative and from negative to positive. Figure 12 shows a time history obtained with R-switching. Comparing Fig. 11 with Fig. 12, we can see that the LR-switching method suppressed vibration better than the R-switching method did. With LR-switching method, equivalent modal damping ratio averaged over the initial three cycles was 5.7%, whereas it was 2.0% with R-switching method. The behavior of V in Figs. 11 and 12 qualitatively coincides with that shown in Fig. 7, indicating that both the energy-recycling method and the R-switching semi-active method work in this actual structure. With the LR-switching energy-recycling method, the change in V at the time of switching is much larger than that with the conventional R-switching method as was expected. This is why LR-switching has higher performance than R-switching does. This behavior of the voltage of the transducer is a result of electric energy-recycling as described in the preceding section. As in the numerical simulation, we can see from Fig. 11 that the collected energy is discharged by irregular switching at around $t = 0.7$ s after a sufficient amount of vibration suppression.

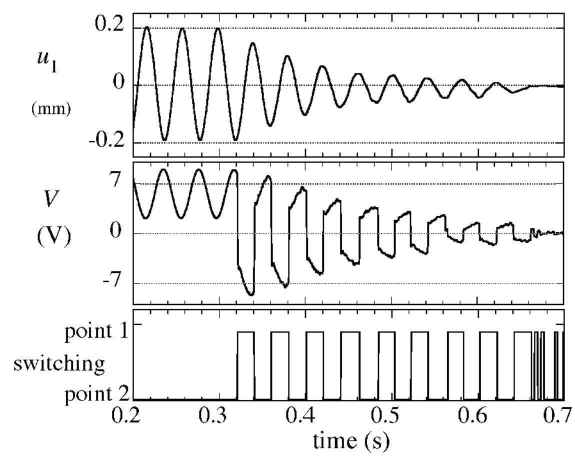


Fig. 11 Time histories of vibration suppression obtained in the experiment by using LR-switching energy-recycling method.

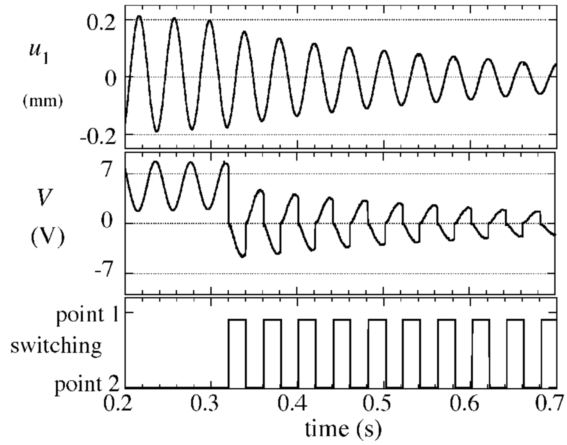


Fig. 12 Time histories of vibration suppression obtained in the experiment by using R-switching semi-active method.

Although the energy-recycling method has been qualitatively shown to be effective in an actual structure, there is some quantitative difference between the results of the numerical simulation and those of the experiment. Comparing Fig. 11 with Fig. 7, we can see that the vibration damped much more slowly in the experiment than in the numerical simulation. Detailed analysis of the time histories reveals that two factors may have caused the slow vibration damping in the experiment. They are apparent differences in $-V_e/V_b$ and $\Delta V/\Delta u_1$, where ΔV is the change in V when u_1 was changed by the amount of Δu_1.

From the time history in Fig. 11, the average value of $-V_e/V_b$ at the first four switchings was 0.558. From Eqs. (2) and (16), we can see that

$$-V_e/V_b \approx e^{-\varsigma_c \pi} \tag{40}$$

Equation (40) suggests that the total resistances in circuits B′ and B was approximately 2.8 Ω. This value is much larger than the resistance of the coil and harness (0.76 Ω), which suggests that the piezoelectric transducer had substantial equivalent resistance. In fact, if we assume that the equivalent resistance of the piezoelectric transducer is 1.44 Ω, that is, the total resistance is 2.2 Ω, the numerical simulation gives $-V_e/V_b = 0.559$. This value of equivalent resistance of the piezoelectric transducer is also much larger than the value of 0.57 Ω, which is the equivalent resistance estimated from the measured loss factor of $\tan\delta = 0.043$ in the voltage–current relationship of the transducer at a frequency of $\omega_c/2\pi = 1.76$ kHz.

Comparing Figs. 7 and 11 further, we can see that the value of $\Delta V/\Delta u_1$ in the experiment was almost two-thirds of the value in the numerical simulation. The resistance against the leakage of the charge, including the voltage measuring system and the circuit shown in Fig. 10, was more than 10 MΩ under static conditions. The load on the transducer per unit Δu_1 also approximately coincided with that in the mathematical model. These results suggest that the effective value of b_a in the vibration suppression experiment was almost two-thirds of the value estimated in the static test. This apparently large equivalent resistance and a small equivalent value of b_a of the piezoelectric transducer in the vibration suppression experiment may have resulted from the effects of transient phenomena, local dynamics in the transducer, and/or dielectric loss of the transducer. A more detailed study of these phenomena is beyond the scope of this paper.

To verify the results, we performed another numerical simulation assuming that the total resistance of the circuit was 2.2 Ω and $b_a = 2.46 \times 10^5$ N/C, which is two-thirds of the value estimated in

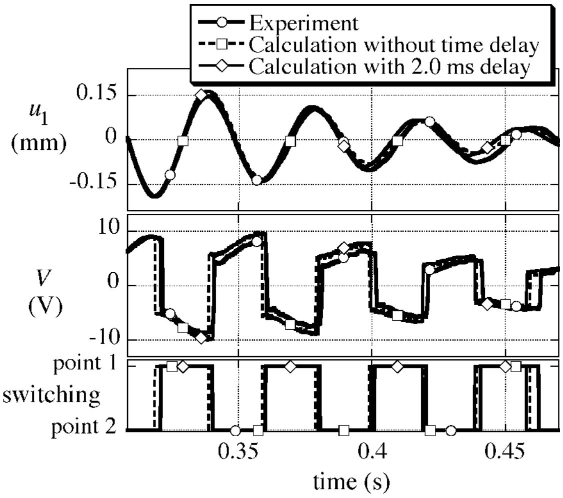


Fig. 13 Time histories obtained in the experiment and in the simulation: $b_a = 2.46 \times 10^5$ N/C, $R = 2.2\ \Omega$, $L = 6.17 \times 10^{-4}$ H, and $W_2 = 1.0 \times 10^3$ kg.

the static test. The 2.0-ms delay in switching in the experiment was also taken into account. Figure 13 shows a comparison of the numerical simulation results with the experimental results. Figure 13 shows that the effect of the 2.0-ms delay on the performance in suppressing vibration is negligible. Figure 13 shows that these values of R and b_a make the simulation results agree well with experimental results. However, as has been mentioned, verification of the phenomena is a subject of future works.

Conclusions

We proposed and investigated a method to improve the performance of semi-active vibration suppression with piezoelectric transducers based on energy recycling that keeps the total system stable. The energy-recycling method collects the electrical energy converted from the mechanical energy of a vibrating structure as an electric charge in the capacitor of the piezoelectric transducer and switches the polarity of the charge to suppress the vibration. By introducing diodes into the electric circuit, we derived a very simple switching law for the energy-recycling method.

To investigate the effectiveness of the energy-recycling method in suppressing vibration, we conducted numerical simulation of vibration suppression of a truss structure by using our energy-recycling semi-active method, a conventional semi-active method, and a passive damper. We compared the results and found that our energy-recycling method can suppress vibration more effectively than both the optimally tuned passive system and the conventional semi-active method. We also found that our method works best when the total resistance of the shunt circuit is relatively small.

To see whether the energy-recycling method works in an actual structure, a preliminary experiment of vibration suppression of a truss was performed by using both the energy-recycling and conventional semi-active methods. The experimental results showed that both methods work well as was expected and that the performance of the energy-recycling method is much higher than that of the conventional semi-active method. The experimental results also showed that the equivalent parameter values in the vibration suppression experiment and those estimated by a static test were slightly different. The equivalent resistance of the piezoelectric transducer was larger than what we had expected, and the equivalent value of the electric–mechanical coupling coefficient b_a was smaller. The reason for this difference is the subject of our future work.

Appendix: Stability of a System with R-Switching and LR-Switching

The total energy of the system described by Eqs. (2), (3), (6), and (7) is

$$E_1(x_1, \dot{x}_1, Q) = \left[k_a x_1^2 + m_1 \dot{x}_1^2 + C_a(-b_a x_1 + Q/C_a)^2\right]/2 \quad \text{(A1)}$$

and that of the system described by Eqs. (2), (3), (14), and (15) is

$$E_2(x_1, \dot{x}_1, Q, \dot{Q}) = \left[k_a x_1^2 + m_1 \dot{x}_1^2 + C_a(-b_a x_1 + Q/C_a)^2 + L\dot{Q}^2\right]/2 \quad \text{(A2)}$$

E_1, E_2, and their derivatives are continuous. E_1 and E_2 are semi-positive definite. They are zero only when all of their arguments are zero. Therefore, E_1 and E_2 are Lyapunov functions if

$$\dot{E}_1 \leq 0, \qquad \dot{E}_2 \leq 0 \quad \text{(A3)}$$

When the switches in Figs. 2 and 3 are kept open or closed, it is clear that Eqs. (A3) hold because the systems are passive and energy dissipative. Because a switching selects either Eq. (6) or (7) for circuit A and either Eq. (14) or (15) for circuit B, and the right-sides of Eqs. (3), (6), (7), (14), and (15) are not infinite, it is clear that the values of $\dot{x}_1$, x_1, and Q do not vary during the infinitesimal duration of switching. This means that the switching does not affect the value of E_1. When the switch of circuit B is turned on, that is, when the system is switched from Eq. (15) to Eq. (14), it is similarly clear from the equations that the value of $\dot{Q}$ does not vary instantaneously. If the switch is turned off when $\dot{Q} = 0$, we can see from Eq. (15) that the value of $\dot{Q}$ does not vary. However, if the switch is turned off when $\dot{Q} \neq 0$, it is clear from Eq. (15) that the value of $\dot{Q}$ jumps to zero, although this is not the case with LR-switching. Therefore, we can see that the value of E_2 decreases if the switch in Fig. 3 is turned off when $\dot{Q} \neq 0$ and that otherwise switching does not affect the value of E_2. Thus, we can conclude that Eqs. (A3) always hold and that the systems with R-switching and LR-switching are stable in the sense of Lyapunov.

There may be a question of where the energy has gone from circuit B when the switch is turned off with $\dot{Q} \neq 0$. To answer this question, it has to be kept in mind that circuit B shown in Fig. 3 is a simplified model of an actual circuit that has inevitable stray capacity. For example, if we consider a small parallel capacitance with the inductor (which is neglected in Fig. 3), we can see from a similar investigation that the switching does not vary the total energy of the system. However, turning off the switch initiates a high-frequency electrical oscillation, and the energy of this oscillation is expected to be dissipated by the inevitable resistance in the circuit in a short period of time.

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